Adaptive Observer Design for Nonlinear Systems Using Generalized Nonlinear Observer Canonical Form

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In this paper, we present an adaptive observer for nonlinear systems that include unknown constant parameters and are not necessarily observable. Sufficient conditions are given for a nonlinear system to be transformed by state-space change of coordinates into an adaptive observer canonical form. Once a nonlinear system is transformed into the proposed adaptive observer canonical form, an adaptive observer can be designed under the assumption that a certain system is strictly positive real. An illustrative example is included to show the effectiveness of the proposed method.

Key Words : Nonlinear System, Adaptive Observer, Nonlinear Observer Canonical Form, Generalized Nonlinear Observer Canonical Form, Strictly Positive Real

1. Introduction

During the last two decades, the area of nonlinear stabilizing controller design has received much attention (Isidori, 1989 ; Nijmeijer and van der Schaft, 1990). Very often the control systems require the measurement of every system state. However, in practical feedback systems all states are not available for feedback because it is impossible to measure some of the states. Even if it is possible to measure the states, several sensors are very expensive. Hence, various output feedback schemes for nonlinear systems have been actively researched by system designers e.g.(Son et al., 2002 ; Choi and Baek, 2002 ; Choi and Kim, 2003). Most of effective techniques for the design of output feedback controller rely on the state observers.

The first systematic design of nonlinear observer was given in (Krener and Isidori, 1983 ; Krener and Respondek, 1985), where the nonlinear observer canonical form was proposed. A simple computation technique for the coordinate transformation of multi-output nonlinear systems was provided in (Xia and Gao, 1989). The block triangular nonlinear observer normal form was proposed by Rudolph and Zeitz (1994) in order to design an observer for more general nonlinear systems. While those approaches are only applicable to a class of observable nonlinear systems, observer design techniques for unobservable systems were proposed by Jo and Seo (2000 ; 2002). In particular, when the system cannot be transformed into the nonlinear observer canonical form, Jo and Seo (2002) constructed a nonlinear observer by transforming the system into the generalized nonlinear observer canonical form.

The goal of this paper is to study the problem of designing adaptive observers for a class of uncertain nonlinear systems. It should be noted that the aforementioned results assume that the system parameters are exactly known. For the design of adaptive observers, Marino (1990) pre-

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sented an observer design method for systems whose nominal system can be transformed into the nonlinear observer canonical form. In contrast to Marino's result, which also requires the assumption that the given system is observable, this paper provides an adaptive observer design scheme for uncertain nonlinear systems that are not necessarily observable. Using the generalized observer canonical form proposed by Jo and Seo (2002), we present sufficient conditions under which an adaptive observer can be designed for uncertain nonlinear systems.

The rest of the paper is organized as follows. In Section 2 the generalized nonlinear observer canonical form in Jo and Seo (2002) is introduced. In addition, Meyer-Kalman-Yakubovic theorem and LaSalle-Yoshizawa theorem are also introduced in this section. Section 3 proposes sufficient conditions under which given nonlinear systems can be transformed into the adaptive observer canonical form which is based on the generalized nonlinear observer canonical form. We also discuss the design method of adaptive observers with this canonical form. Section 4 is devoted to illustrate the proposed design method with a numerical example and finally some conclusions are given in Section 5.

Before we begin, some notations used in the paper are to be specified:

- a Hurwitz matrix is a matrix whose eigenvalues have all their negative real parts
- a Hurwitz polynomial is a polynomial whose roots have all their negative real parts
- a symmetirc matrix $A \in R^{n\times n}$ is said to be positive definite if $x^TAx > 0$ for all $x \in R^n$, $x \neq 0$
- a system $W(s)$ that satisfies $W(jw) > 0$, $\forall w \in R$ is said to be strictly positive real (SPR)
- a vector field f is said to be complete if the solutions to the differential equation $\dot{x} = f(x)$ is defined for all $t \in R$
- a function $V : R^n \to R^+$ is said to be radially unbounded if $V(x) \to \infty$ as $|x| \to \infty$
- Lie Bracket of vector fields f and g ($f, g : R^n \to R^n$) are defined by

$$[f, g] := \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g$$

- the Jacobian matrix of $f(x_1, x_2)$ with respect to its first and second argument at (x_1, x_2) are denoted by $D_1 f(x_1, x_2)$, $D_2 f(x_1, x_2)$, respectively.

For the stability definitions such as globally uniformly bounded and globally uniformly asymptotically stability, the reader is referred to the reference (Isidori, 1989) for details.

2. Generalized Nonlinear Observer Canonical Form

In this paper we shall consider single output nonlinear systems with unknown constant parameters θ_i

$$\begin{aligned} \dot{x} &= f(x) + g(x, u) + \sum_{i=1}^{p} \theta_i q_i(x, u) \\ y &= h(x) \end{aligned} \tag{1}$$

in which $x \in R^n$, $u \in R^m$, $y \in R^1$, $\theta = [\theta_1, \cdots, \theta_p]^T \in R^p$, $f(0) = 0$, $h(0)$, $g(x, 0) = 0$, $\forall x \in R^n$, $q_i(0, u) = 0$, $1 \le i \le p$, $\forall u \in R^r$. We first consider the case where no uncertainties are present in nonlinear systems (1), i.e., $\theta = 0$. When there are no uncertainties, most of research result on nonlinear observers (Krener and Isidori, 1983; Krener and Respondek, 1985; Xia and Gao, 1989; Rudolph and Zeitz, 1994) assumed that nonlinear system (1) should be observable, which, in turn, requires

$$\text{rank}\{dh, \cdots, dL_f^{n-1}h\} = n \tag{2}$$

In order to design an observer for nonlinear systems that do not satisfy (2), Jo and Seo (2002) proposed the generalized nonlinear observer canonical form. Let $r (\le n)$ be the largest integer that satisfy the followings:

$$\text{rank}\{dh, \cdots, dL_f^{i-1}h\} = i \tag{3}$$

Definition 2.1 The following nonlinear system is called generalized nonlinear observer canonical form:

$$\begin{aligned} \dot{z}_o &= A_o z_o + \gamma_o(y, u) \\ & \quad z_o \in R^r,\ A_o \in R^{r\times r} \\ \dot{z}_{\bar{o}} &= A_{\bar{o}o} z_o + f_{\bar{o}}(y, z_{\bar{o}}) + \gamma_{\bar{o}}(y, y) \\ & \quad z_{\bar{o}} \in R^{n-r},\ A_{\bar{o}o} \in R^{(n-r)\times r} \end{aligned} \tag{4}$$

$$y=C_o z_o,\ C_o \in R^r$$

where

$$A_o=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix},\ C_o=[0\ 0\ \cdots\ 0\ 1]$$

Once the given nonlinear system is transformed into generalized nonlinear observer canonical form, the state observer can be designed easily (Jo and Seo, 2002). In order to introduce the conditions under which such transformation exists, we need the following: Given vector field τ, the vector field $X_i(\tau)$, $1 \le i \le r$ is defiend by

$$X_i(\tau)=ad_{(-f)}^{i-1}\tau,\ 1 \le i \le r \tag{5}$$

When there is no confusion, we will denote $X_i(\tau)$ by X_i throughout the paper.

Theorem 2.2 (Jo and Seo, 2002) There exists a global diffeomorphism

$$z=T(x),\ T(0)=0,\ z\in R^n$$

transforming the system (1) into the generalized nonlinear observer canonical form (4) if there exists a vector fields τ, X_{r+1}, $\cdots$, X_n such that:

i) $\langle dL_f^i h,\ \tau\rangle=\delta_{i,r-1}$, $0 \le i \le r-1$

ii) $L_{X_i}h=0$, $r+1 \le i \le n$

iii) $\mathrm{rank}\{X_1, \cdots, X_n\}=n$

iv) $[X_i, X_j]=0$, $1 \le i, j \le n$

v) $[g, X_j]=0$, $1 \le j \le n$, $j \ne r$

vi) $[f, X_i] \in \mathrm{span}\{X_{r+1}, \cdots, X_n\}$, $r+1 \le i \le n$

vii) $ad_f^i \tau$, $0 \le i \le r-1$ are complete vector fields

Using Theorem 2.2, we can design an observer for nonlinear systems with no uncertainties. However, for nonlinear system (1) with unknown parameters θ, Theorem 2.2 cannot be used and an adaptive observer should be designed that takes uncertainties into the consideration.

Definition 2.3 A global adaptive observer for system (1) is

$$\begin{aligned} &\dot{w}=\alpha_1(w,\hat{\theta},y,u),\ w(0)=w_0,\ w\in R^l,\ l\ge n \\ &\dot{\hat{\theta}}=\alpha_2(w,\hat{\theta},y,u),\ \hat{\theta}(0)=\hat{\theta}_0,\ \hat{\theta}\in R^p \\ &\hat{x}=\alpha_3(w,\hat{\theta},y,u),\ \hat{x}\in R^n \end{aligned} \tag{6}$$

that satisfies the conditions (i) and (ii) for every $x(0)\in R^n$, $w_0 \in R^l$, $\hat{\theta}_0 \in R^p$ for any value of the unknown parameter θ, and for bounded $\|x(t)\|$, $\|u(t)\|$, $\forall t \ge 0$:

(i) $\|w(t)\|$, $\|\hat{\theta}(t)\|$, $\|x(t)-\hat{x}(t)\|$ are bounded for all $t \ge 0$

(ii) $\lim_{t\to\infty}\|x(t)-\hat{x}(t)\|=0$

In order to design a global adaptive observer, we need the following result.

Theorem 2.4 (Meyer, 1965) - Meyer-Kalman-Yakubovic (MKY) Let $A\in R^{n\times n}$ be a Hurwitz matrix and b, c^T are $n\times 1$ vectors. The triple (A, b, c) satisfies the strictly positive real condition

$$\begin{gathered} Re\{c(jwI-A)^{-1}b\}>0 \\ \forall w \in (-\infty, +\infty) \end{gathered} \tag{7}$$

if and only if, for given positive definite matrix Q, there exist a positive definite matrix $P\in R^{n\times n}$, vector $q\in R^{n\times 1}$ and a constant $\varepsilon>0$ such that

$$\begin{gathered} A^T P+PA=-qq^T-\varepsilon Q \\ Pb=c^T \end{gathered}$$

3. Design of Adaptive Observer

In order to design an adaptive observer for the generalized nonlinear observer canonical form (4), adaptive observer canonical form is defined as follows:

$$\begin{aligned} &\dot{z}_o=A_o z_o+\gamma_o(y,u)+b\sum_{i=1}^{p}\theta_i\beta_i(y,u) \\ &\qquad z_o\in R^r,\ A_o\in R^{r\times r},\ b\in R^r,\ \beta_i: R^1\times R^m \to R^1 \\ &\dot{z}_{\bar{o}}=A_{\bar{o}o}z_o+f_{\bar{o}}(y,z_{\bar{o}})+\gamma_{\bar{o}}(y,y) \\ &\qquad z_{\bar{o}}\in R^{n-r},\ A_{\bar{o}o}\in R^{(n-r)\times r} \\ &y=C_o z_o,\ C_o\in R^r \end{aligned} \tag{9}$$

where, A_o, C_o are defined in Definition 2.1. If we define

$$\beta(y,u)=[\beta_1(y,u),\ \cdots,\ \beta_n(y,u)]^T$$

then system (9) can be represented as follows:

$$\begin{aligned} \dot{z}_o &= A_o z_o + \gamma_o(y, u) + b\beta^T(y, u)\theta \\ \dot{z}_{\bar{o}} &= A_{\bar{o}o} z_o + f_{\bar{o}}(y, z_{\bar{o}}) + \gamma_{\bar{o}}(y, u) \\ y &= C_o z_o \end{aligned} \tag{10}$$

Now, we are able to present sufficient conditions under which nonlinear systems (1) can be transformed into adaptive observer canonical form (10).

Theorem 3.1 There exists a global diffeomorphism $z=T(x)$, $T(0)=0$, $z\in R^n$ transforming the system (1) into the adaptive observer canonical form (10) if there exist smooth vector fields τ, $X_{r+1}, \cdots, X_n$ such that:

i) all assumptions of Theorem 2.2 are satisfied,

ii) $[q_i, X_j]=0,\ 1\le i\le p,\ 1\le j\le n,\ j\ne r$

iii) $[X_r, q_i]=\alpha_i(y, y)\sum_{j=1}^{r} b_j X_j,\ 1\le i\le p,\ \forall u\in R^r$,

where $\beta_i(y, u)=\int_0^y \alpha_i(\zeta, u)\,d\zeta$.

Proof: It follows from assumptions (ii), (iv), (vii) of Theorem 2.2 and Simultaneous rectification theorem (Nijmeijer and van der Schaft, 1990) that there exist a global diffeomorphism $z=T(x)=(t_1(x), \cdots, t_n(x))$ which satisfies

$$\left\langle \begin{bmatrix} dt_1 \\ \vdots \\ dt_n \end{bmatrix}, [X_1, \cdots, X_n] \right\rangle = I_n \tag{11}$$

that is, in new coordinates

$$\begin{aligned} ad^i_{(-f)}\tau &= \frac{\partial}{\partial z_{i+1}},\ 0\le i\le r-1 \\ X_i &= \frac{\partial}{\partial z_i},\ r+1\le i\le n \end{aligned} \tag{12}$$

Let $f=\bar{f}_1\frac{\partial}{\partial z_1}+\cdots+\bar{f}_n\frac{\partial}{\partial z_n}$. From (12), we have

$$\begin{aligned} ad^i_{(-f)}\tau &= [ad^{j-1}_{(-f)}\tau, f]=\left[\frac{\partial}{\partial z_j}, \bar{f}\right] \\ &= \sum_{i=1}^{n}\left(\frac{\partial}{\partial z_i}\bar{f}_i\right)\frac{\partial}{\partial z_i}=\frac{\partial}{\partial z_{j+1}} \end{aligned}$$

which implies that

$$\begin{aligned} &\frac{\partial}{\partial z_j}\bar{f}_i=0,\ 1\le i\le n,\ i\ne j+1,\ 1\le j\le r-1 \\ &\frac{\partial}{\partial z_j}\bar{f}_{j+1}=1,\ 1\le j\le r-1 \end{aligned} \tag{13}$$

Moreover, from assumption (vi) of Theorem 2.2, we have

$$\begin{aligned} [f, X_j] &= \left[\bar{f}, \frac{\partial}{\partial z_j}\right]=\sum_{i=1}^{l}\left(\frac{\partial}{\partial z_j}\bar{f}_i\right)\frac{\partial}{\partial z_i} \\ &\in \mathrm{span}\left\{\frac{\partial}{\partial z_{r+1}}, \cdots, \frac{\partial}{\partial z_n}\right\} \end{aligned}$$

which implies that

$$\frac{\partial}{\partial z_j}\bar{f}_i=0,\ 1\le i\le r,\ r+1\le j\le n \tag{14}$$

From (13) and (14), we get

$$\begin{aligned} f_1 &= \bar{f}_1(z_r) \\ f_2 &= \bar{f}_2(z_r)+z_1 \\ &\vdots \\ f_r &= \bar{f}_r(z_r)+z_{r-1} \\ f_{r+1} &= \bar{f}_{r+1}(z_r, z_{r+1}, \cdots, z_n) \\ &\vdots \\ f_n &= \bar{f}_n(z_r, z_{r+1}, \cdots, z_n) \end{aligned}$$

Now, we determine the components of the vector field $g(x, u)$ in the new coordinates $g=\sum_{i=1}^{n}\bar{g}(z, u)\frac{\partial}{\partial z_i}$. From (12) and assumption (v) of Theorem 2.2, we have

$$\begin{aligned} [ad^j_f\tau, g] &= \left[\frac{\partial}{\partial z_{j+1}}, \bar{g}\right] \\ &= \sum_{i=1}^{n}\left(\frac{\partial}{\partial z_{j+1}}\bar{g}_i\right)\frac{\partial}{\partial z_i}=0,\ 0\le j\le r-2 \end{aligned}$$

which implies that

$$\frac{\partial}{\partial z_{j+1}}\bar{g}_i=0,\ 1\le i\le n,\ 0\le j\le r-2 \tag{15}$$

Moreover, from assumption (v) of Theorem 2.2, we have

$$\begin{aligned} [g, X_j] &= \left[\bar{g}, \frac{\partial}{\partial z_j}\right] \\ &= \sum_{i=1}^{l}\left(\frac{\partial}{\partial z_j}\bar{g}_i\right)\frac{\partial}{\partial z_i}=0,\ r+1\le j\le n \end{aligned}$$

which implies that

$$\frac{\partial}{\partial z_j}\bar{g}_i=0,\ 1\le i\le n,\ r+1\le j\le n \tag{16}$$

Thus, it follows from (15) and (16) that

$$g_i=\bar{g}_i(z_r, u, 1\le i\le n) \tag{17}$$

In a similar manner, from assumption (ii), we have

$$q_{ji}=\bar{q}_{ji}(z_r, u),\ 1\le i\le n,\ 1\le j\le p \tag{18}$$

In addition, assumption (iii) leads to (for $i=1$)

$$\left[\frac{\partial}{\partial z_r}, \bar{q}_1(z, u)\right]=\alpha_1(z_n, u)\sum_{i=1}^{r} b_i \frac{\partial}{\partial z_i} \tag{19}$$

which implies, together with (18)

$$\sum_{i=1}^{n}\left(\frac{\partial}{\partial z_r}\bar{q}_{1i}(z_r, u)\right)\frac{\partial}{\partial z_i}=\alpha_1(z_r, u)\sum_{i=1}^{r} b_i \frac{\partial}{\partial z_i}$$

Thus, we have

$$\bar{q}_{1i}(z_r, u)=\begin{cases}\int_0^{z_r}\alpha_1(\zeta, u)\, b_i d\zeta + C_i, & 1\le i\le r\\ C_i, & r+1\le i\le n\end{cases}$$

where C_i is an appropriate integration constant. Since $T(0)=0$, $q_1(0, u)=0$, we have

$$\bar{q}_{1i}(z_r, u)=\begin{cases} b_i\int_0^{z_r}\alpha_1(\zeta, u)\, d\zeta, & 1\le i\le r\\ 0, & r+1\le i\le n\end{cases} \tag{20}$$

which implies, together with the definition of $\beta_i(y, u)$

$$\bar{q}_1(z_r, u)=\beta_1(z_r, u)\sum_{i=1}^{r} b_i \frac{\partial}{\partial z_i}$$

For $j=2, \cdots, p$, we can similarly get

$$\bar{q}_j(z_r, u)=\beta_j(z_r, u)\sum_{i=1}^{r} b_i \frac{\partial}{\partial z_i},\ 1\le j\le p$$

By virtue of Leibniz's formula and the definition of τ

$$\langle dh, ad_{(-f)}{}^i\tau\rangle=\frac{\partial h}{\partial z_{i+1}}=0,\ 0\le i\le r-2$$

$$\langle dh, ad_{(-f)}^{r-1}\tau\rangle=\frac{\partial h}{\partial z_r}=1$$

and assumption (ii) of Theorem 2.2, we have

$$L_{X_j}h=\left\langle dh, \frac{\partial}{\partial z_j}\right\rangle=\frac{\partial h}{\partial z_j}=0,\ r+1\le j\le n$$

Because of $h(0)=0$, we have $y=h(x)=zr$. In summary, defining

$$\gamma_{oi}(y, u)=\bar{f}_i(y)+\bar{g}_i(y, u),\ 1\le i\le r$$

$$\gamma_{\bar{o}i}(y, u)=\bar{g}_i(y, u),\ r+1\le i\le n$$

system (1) can be expressed in the new coordinates as follows:

$$\dot{z}_1=\gamma_{o1}(y, u)+b_1\sum_{j=1}^{p}\theta_j\beta_j(y, u)$$

$$\dot{z}_i=z_{i-1}+\gamma_{oi}(y, u)+b_i\sum_{j=1}^{p}\theta_j\beta_j(y, u),\ 2\le i\le r$$

$$\dot{z}_i=\bar{f}_{\bar{o}}(z_{r+1}, \cdots, z_n)+\gamma_{\bar{o}i}(y, u),\ r+1\le i\le n$$

Now, we investigate the conditions under which there exist an adaptive observer for the adaptive observer canonical form (10)

Theorem 3.2 Suppose that the following conditions are met for the system (10):

(i) There exist a positive constant $k_0>0$ and a positive definite matrix $P_2\in R^{(n-r)\times(n-r)}$ such that

$$v^T P_2[D_2 f_{\bar{o}}(y, z_{\bar{o}})]v\le -k_0\|v\|^2 \quad \forall (y, z_{\bar{o}}, v)\in R^r\times R^{n-r}\times R^{n-r}$$

(ii) There exist a Hurwitz polynomial $k_1+k_2 s+\cdots+k_r s^{r-1}+s^r$ such that

$$W(s)=\frac{b_1+b_2 s+\cdots b_r s^{r-1}}{k_1+k_2 s+\cdots+k_r s^{r-1}+s^r}$$

is strictly positive real.

Then, for every bounded input $u(t)$, the system (21) is a global adaptive observer for system (10).

$$\begin{aligned}\dot{\hat{z}}_o&=A_o\hat{z}_o+\gamma_o(y, u)+b\beta^T(y, u)\hat{\theta}+K(y-\hat{z}_r)\\ \dot{\hat{z}}_{\bar{o}}&=A_{\bar{o}\bar{o}}\hat{z}_o=f_{\bar{o}}(y, \hat{z}_{\bar{o}})+\gamma_{\bar{o}}(y, u)\\ \dot{\hat{\theta}}&=\Gamma\beta(y, u)(y-\hat{z}_r)\end{aligned} \tag{21}$$

where $K=[k_1, k_2, \cdots, k_r]^T$ is given in assumption (ii) and Γ is an arbitrary symmetric positive definite matrix.

Proof: Without loss of generality, we assume $p=1$. Let the error signal be defined by $e_o=\hat{z}_o-z_o$, $e_{\bar{o}}=\hat{z}_{\bar{o}}-z_{\bar{o}}$, and $e_\theta=\hat{\theta}-\theta$. Then, it follows from (10) and (21) that $\dot{e}_o=(A_o-KC_o)e_o+e_\theta\beta(y, u)b$. Since $k_1+k_2s+\cdots+k_r s^{r-1}+s^r$ is the characteristic polynomial of the matrix A_o-KC_o, it follows from assumption (ii) that matrix A_o-KC_o is Hurwitz. Since $W(s)$ is realized by the triple (A_o-KC_o, b, C_0) and since $W(s)$ is strictly positive real, Meyer-Kalman-Yacubovich theorem can be applied: Given a positive definite matrix L, and a scalar $\mu>0$, there exist a positive definite matrix P and a vector r such that

$$(A_o-KC_o)^TP+P(A_o-KC_o) = -rr^T-\mu L=:-Q$$
$$Pb=C_o^T$$

Now, consider the following Laypunov function

$$V_1(e_o, e_\theta)=e_o^TPe_o+e_\theta^T\Gamma^{-1}e_\theta$$

The time derivative of V_1 is

$$\begin{aligned}\dot{V}&=-e_o^TQe_o+2e_\theta(e_o^TPb)\beta(y, u)+2e_\theta\Gamma^{-1}\dot{e}_\theta\\&=-e_o^TQe_o+2e_\theta[(e_o^TPb)\beta(y, u)+\Gamma^{-1}\dot{e}_\theta]\end{aligned}$$

Since $\dot{e}_\theta=\dot{\hat{\theta}}$ and $Pb=C_o^T$, we have

$$\begin{aligned}\dot{V}&=-e_o^TQe_o+2e_\theta[(e_o^TC_o^T)\beta(y, u)+\Gamma^{-1}\dot{\hat{\theta}}]\\&=-e_o^TQe_o+2e_\theta[(\hat{z}_r-y)\beta(y, u)+\Gamma^{-1}\dot{\hat{\theta}}]\\&=-e_o^TQe_o\leq 0\end{aligned}$$

which, in turn, implies that e_o and e_θ are globally uniformly bounded, i.e., $\| \hat{z}_o(t)-z_o(t)\|$ and $\| \hat{\theta}(t)-\theta(t)\|$ are bounded for all $t\geq 0$. Moreover, from LaSalle-Yoshizawa theorem (LaSalle, 1968; Yoshizawa, 1966), we have $\lim_{t\to\infty} e_o^T(t) Qe_o(t)=0$. Since Q is a positive definite matrix, we can guarantee that

$$\lim_{t\to\infty} e_o(t)=0 \tag{22}$$

Now, for the convergence analysis of $e_{\bar{o}}$, consider the following Lyapunov function

$$V_2(e_{\bar{o}})=\frac{1}{2}e_{\bar{o}}^TP_2e_{\bar{o}}$$

where $P_2\in R^{(n-r)\times(n-r)}$ is given by assumption (i). From (10) and (21), we have

$$\dot{e}_{\bar{o}}=A_{\bar{o}o}e_o+f_{\bar{o}}(y, \hat{z}_{\bar{o}})-f_{\bar{o}}(y, z_{\bar{o}})$$

The time derivative of V_2 is

$$\dot{V}=e_{\bar{o}}^TP_2[A_{\bar{o}o}e_o+f_{\bar{o}}(y, \hat{z}_{\bar{o}})-f_{\bar{o}}(y, z_{\bar{o}})]$$

Mean-value theorem and assumption (i) yield to

$$\begin{aligned}\dot{V}_2&=e_{\bar{o}}^TP_2[A_{\bar{o}o}e_o+D_2f_{\bar{o}}(y, \tilde{z}_{\bar{o}})e_{\bar{o}}]\\&\leq -k_0\| e_{\bar{o}}\|^2+e_{\bar{o}}^TP_2A_{\bar{o}o}e_o\\&\leq -\frac{1}{2}k_0\| e_{\bar{o}}\|^2-\frac{1}{2}k_0\left(\| e_{\bar{o}}\|-\frac{\| P_2A_{\bar{o}o}\|}{k_0}\| e_o\|\right)^2\\&\quad+\frac{\| P_2A_{\bar{o}o}\|^2}{2k_0}\| e_o\|^2\\&\leq -\frac{1}{2}k_0\| e_{\bar{o}}\|^2+\frac{\| P_2A_{\bar{o}o}\|^2}{2k_0}\| e_o\|^2\end{aligned} \tag{23}$$

where $\tilde{z}_{\bar{o}}\in\{t\hat{z}_{\bar{o}}+(1-t)z_{\bar{o}}:0\leq t\leq 1\}$. Since it has been already shown that e_o is bounded and $\lim_{t\to\infty} e_o(t)=0$, (23) implies that (24)

$$\lim_{t\to\infty} e_{\bar{o}}(t)=0 \tag{24}$$

Therefore, it can be seen from (22) and (24) that system (21) is a global adaptive observer for system (10).

At first glance, one might think that the assumption (i) of Theorem 3.2 can be satisfied with only very limited class of nonlinear systems. But, as is shown in the work of Jo and Seo (2000), it is weaker than requiring that

1) $f_{\bar{o}}(y, z_{\bar{o}})=A_{\bar{o}}z_{\bar{o}}+\bar{\gamma}(y)$, and
2) $A_{\bar{o}}$ is an arbitrary Hurwitz matrix

where $\bar{\gamma}(\cdot)$ is an arbitrary function. So, assumption (i) is similar to the detectability condition in the linear systems theory. Moreover, it seems that the assumption (ii) of Theorem 3.2 is rather restrictive. But, it is more general form than the simple condition that the polynomial $b_rs^{r-1}+\cdots+b_1$ is Hurwitz. The following corollary express the assumption (ii) of Theorem 3.2 in much simpler form.

Corollary 3.3 Suppose that the following conditions are met for system (10):

(i) assumption (i) of Theorem 3.2 is satisfied.
(ii) for the vector $b=[b_1, \cdots, b_r]^T$, the polynomial $b_rs^{r-1}+\cdots+b_1$ is Hurwitz and $b_r>0$.

Then, for every bounded input $u(t)$, the system (21) is a global adaptive observer for system (10), where the observer gain is given by

$$K=\frac{1}{b_r}(A_0b+\lambda b) \tag{25}$$

with an arbitrary constant $\lambda>0$.

Proof: Defining error signal as in the proof of Theorem 3.2, we have

$$\dot{e}_o=(A_o-KC_o)e_o+b\beta^T(y, u)e_\theta$$

From $K=[k_1, \cdots, k_r]^T$ and (25), we have

$$b_r b_1 = \lambda b_1$$
$$\vdots$$
$$b_r k_{r-1} = b_{r-2} + \lambda b_{r-1}$$
$$b_r k_r = b_{r-1} + \lambda b_r$$

which, in turn, implies that

$$\begin{aligned} & b_r(s^r + k_r s^{r-1} + \cdots + k_1) \\ & = (s+\lambda)(b_r s^{r-1} + \cdots + b_1) \end{aligned} \tag{26}$$

Since $\lambda > 0$ and the polynomial $b_r s^{r-1} + \cdots + b_1$ is Hurwitz, the polynomial $s^r + k_r s^{r-1} + \cdots + k_1$ is also Hurwitz. and hence the matrix $A_0 - KC_0$ is Hurwitz. From (26), we obtain

$$\begin{aligned} C_o(sI - A_o)^{-1} b &= \frac{b_r s^{r-1} + \cdots + b_1}{s^r + k_{r-1} s^{r-1} + \cdots + k_1} \\ &= \frac{b_r}{s+\lambda} \end{aligned}$$

which, in turn, implies

$$Re[C_o(jwI - A + KC_o)^{-1} b] = \frac{b_r \lambda}{\lambda^2 + w^2} > 0$$

Thus, the triple $(A_o - KC_o, b, C_o)$ satisfies the strict positive real condition (7) and hence Meyer-Kalman-Yacubovich theorem can be applied. The remainder of the proof is similar to that of Theorem 3.2 and hence omitted.

According to Theorem 3.1 and Corollary 3.3, we have the following main theorem.

Theorem 3.4 Suppose that the following conditions are met for system (1):

(i) all assumptions of Theorem 3.1 are satisfied.

(ii) all assumptions of Corollary 3.3 are satisfied.

Then, system (27) is a global adaptive observer for system (1):

$$\begin{aligned} &\hat{x} = T^{-1}([\hat{z}_o \; \hat{z}_{\bar{o}}]^T) \\ &\dot{\hat{z}}_o = A_o \hat{z}_o + \gamma_o(y, u) + b\beta^T(y, u)\hat{\theta} + K(y - \hat{z}_r) \\ &\dot{\hat{z}}_{\bar{o}} = A_{\bar{o}o}\hat{z}_o + f_{\bar{o}}(y, \hat{z}_{\bar{o}}) + \gamma_{\bar{o}}(y, u) \\ &\dot{\hat{\theta}} = \Gamma\beta(y, u)(y - \hat{z}_r) \\ &K = \frac{1}{b_r}(A_o b + \lambda b) \end{aligned} \tag{27}$$

where λ is an arbitrary positive constant and Γ is an arbitrary symmetric positive definite matrix.

4. An Illustrative Example

In order to illustrate the proposed adaptive observer we consider the following nonlinear system

$$\begin{aligned} &\dot{x}_1 = x_2 - x_1^3 + u + \theta x_1 u \\ &\dot{x}_2 = -x_1^2 x_2 + x_1 \sin x_1 u + 2\theta x_1 u - \theta x_1^3 u \\ &\dot{x}_3 = -x_3 - x_3^3 + x_1 x_2 + x_1 u + \theta x_1^2 u \\ &y = x_1 \end{aligned} \tag{28}$$

Since

$$\begin{aligned} dh &= [1\ 0\ 0]^T \\ d(L_f h) &= [-3x_1^2\ 1\ 0]^T \\ d(L_f^2 h) &= [15x_1^4 - 8x_1 x_2\ \ -4x_1^2\ \ 0]^T \end{aligned}$$

the system (28) does not satisfy the condition (2), and hence it is not observable. Now, we let $r=2$ so that (3) is satisfied and select $\tau = \frac{\partial}{\partial x_2}$ so that the condition (i) of Theorem 3.1 holds. According to (5), we have $X_1 = \frac{\partial}{\partial x_2}$, $X_2 = \frac{\partial}{\partial x_1} - x_1^2 \frac{\partial}{\partial x_2} + x_1 \frac{\partial}{\partial x_3}$, and X_3 can be chosen as $X_3 = \frac{\partial}{\partial x_3}$ so that the conditions (iii) and (iv) are met. Since

$$q = \begin{bmatrix} x_1 u \\ 2x_1 u - x_1^3 u \\ x_1^2 u \end{bmatrix}$$

it can be seen that the assumptions (i), (ii), (v), (vi), and (viii) of Theorem 3.1 is satisfied. Moreover, since

$$\begin{aligned} [X_2, q] &= \begin{bmatrix} u \\ -x_1^2 u + 2u \\ x_1 u \end{bmatrix} \\ &= u(2X_1 + X_2) \end{aligned} \tag{29}$$

it is easily seen that the condition (vii) of Theorem 3.1 is met with $b = [2\ 1]^T$, $\alpha(y, u) = u$, and $\beta(y, u) = yu$. From (11) we obtain

$$T(x) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_1^3/3 + x_2 \\ x_1 \\ -x_1^2/2 + x_3 \end{bmatrix} \tag{30}$$

Using the transformation (30), the system (28) can be expressed in the new coordinate as follows:

$$\begin{aligned}\dot{z}_1&=-y^5+(y\sin y+y^2)u+2\theta yu\\ \dot{z}_2&=z_1-\frac{4}{3}y^3+u+\theta yu\\ \dot{z}_3&=-z_3-\left(z_3+\frac{1}{2}y^2\right)^3-\frac{1}{2}y^2+y^4\\ y&=z_2\end{aligned}\tag{31}$$

It can be seen that the condition (i) of Corollary 3.3 is met with $P_2=1$, $k_0=1$. From $b=[2\ \ 1]^T$, the polynomial $s+2$ is Hurwitz, and hence the condition (ii) of Corollary 3.3 are satisfied. According to Theorem 3.4, an adaptive observer for the system (28) is given by

$$\hat{x}=\begin{bmatrix}\hat{z}_2\\ \hat{z}_1-\hat{z}_2^3/3\\ \hat{z}_3+\hat{z}_2^2/2\end{bmatrix}$$

$$\begin{aligned}\dot{\hat{z}}_1&=-y^5+(y\sin y+y^2)u+2\hat{\theta}yu+k_1(y-\hat{z}_2)\\ \dot{\hat{z}}_2&=z_1-\frac{4}{3}y^3+u+\hat{\theta}yu+k_2(y-\hat{z}_2)\\ \dot{\hat{z}}_3&=-\hat{z}_3-\left(z_3+\frac{1}{2}y^2\right)^3-\frac{1}{2}y^2+y^4\\ \dot{\hat{\theta}}&=\Gamma yu(y-\hat{z}_2)\end{aligned}$$

From (25), observer gain is given by

$$K=\begin{bmatrix}k_1\\ k_2\end{bmatrix}=\begin{bmatrix}2\lambda\\ 2+\lambda\end{bmatrix}$$

where λ is an arbitrary positive constant. In addition, according to Theorem 3.4, the parameter Γ can be chosen as any positive constant. The simulations were performed using MATLAB and results are shown in Fig. 1 under the following conditions: $u=\sin(\pi t/3)*\sin(\pi t/2)$, $\lambda=2$, $\Gamma=5$, $\theta=1$, $x(0)=\begin{bmatrix}0.5\\ 1\\ -1\end{bmatrix}$, $\hat{z}(0)=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$, $\hat{\theta}(0)=0$.

It can be seen from figure that the proposed adaptive observer estimates the true state after approximately 2 seconds.

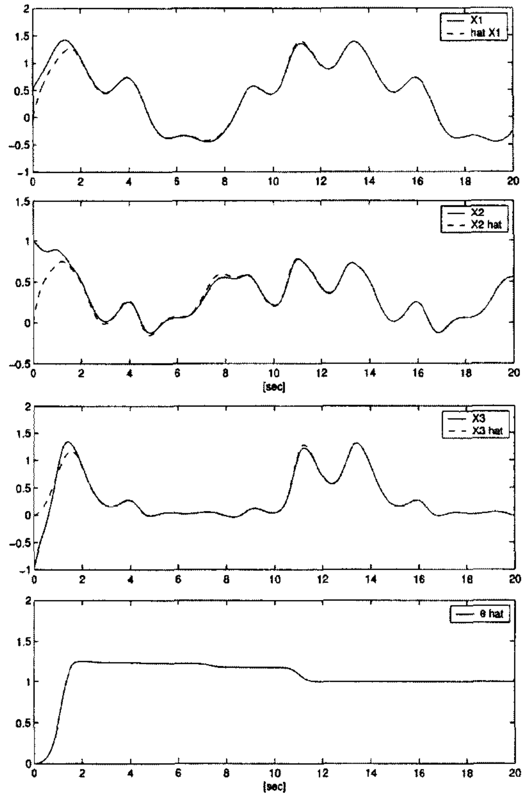

Fig. 1 Simulation Results

5. Conclusion

This paper has presented an adaptive observer for nonlinear systems using the generalized nonlinear observer canonical form. We have presented sufficient conditions for a nonlinear system to be transformed into the adaptive observer canonical form. Sufficient conditions in terms of strictly positive real conditions have also been presented under which an adaptive observer exists for the proposed canonical form. In addition, the stability of the proposed adaptive observer has been verified through the computer simulation. A possible improvement of the proposed observer could be made by weakening the assumptions needed in the design of the proposed scheme.

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